FLEXURAL BUCKLING OF FRAMES ACCORDING TO THE NEW EC3-RULES – A COMPARATIVE, PARAMETRIC STUDY

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Abstract. This paper describes the application of the specific methods of the new EN 1993-1-1 for the structural design of steel frames. The methods investigated are based on geometrically, materially nonlinear analyses of the imperfect structure (GMNIA), on second-order elasto-plastic analyses and on the two methods using member design formulae. The one is related to non-sway buckling lengths, the other is the equivalent column method.

1 INTRODUCTION

The present EN 1993-1-1 specifies several methods for the calculation of the flexural buckling behaviour of steel frames. Depending on the accuracy of the intended results there are first-order and second-order elastic and plastic methods available. The detrimental effects of geometrical imperfections and residual stress always have to be accounted for. In some analysis types they have to be introduced directly into the model, in cases when the beam-column formulae is used member imperfections and residual stress are already accounted for, however, sway imperfections have to be applied additionally. When the equivalent column method is used, all imperfections may be regarded to be covered by the buckling checks based on effective column lengths. The present paper aims at comparing the different design methods with each other in order to get a thorough understanding of the behaviour of each method. The comparisons are performed numerically and are illustrated by interaction diagrams.

Results presented in detail are those for GMNIA performed by ABAQUS [1] and those of several elasto-plastic first-order and second-order analyses. Results of the method using "cut-out-members" with second-order end moments and non-sway buckling lengths (alternatively full length) are illustrated and compared with each other. The study also illustrates results of the equivalent column method using appropriate buckling lengths according to the global buckling mode of the frames. Furthermore, a comparison with the Amplified Sway Method of the British Standard BS 5950-1:2000 is included. At the end of the paper it is shown, that with a modified equivalent column method rather accurate results could be achieved.

The methods were applied to a hinged sway frame with three different load cases and a clamped sway frame with two different load cases. There are pure symmetrical (non-sway) or pure antimetrical (sway) load cases. For the hinged sway frame the behaviour of a half sway and half non-sway load case is also investigated. The results are illustrated by interaction diagrams depending on the parameters $N$, $M_y$ and the column slenderness $\lambda_v$.

By these load cases the complex interaction behaviour between second-order effects due to sway and non-sway deflections has been studied. Thus, a wide range of possible frame systems is covered.
2 REVIEW ON EXISTING RESEARCH ON THE CORRECT APPLICATION OF THE C_{my}-FACTOR FOR EQUIVALENT MEMBERS

The stability-check of sway frames may be based on the buckling check of individual members, which are considered as isolated columns cut-out of the system and are loaded by the end forces/moments of the structure – apart from internal loads along the span - (called COM). These end moments have to include the second-order and imperfection-effects of the sway-mode (P.∆-effect) and in some cases the effects of the local imperfections according to EN 1993-1-1 (P.δ-effect), confer Fig. 1 and Fig. 7.

Alternatively, buckling-checks may be based on the equivalent column method (called ECM). It is applied here in its traditional form as pin-ended column (Euler-case II) of uniform cross-section and uniform axial force with a buckling length, which leads to the same critical buckling capacity N_{cr} as the structure considered [7][8].

It has been noticed in several papers [2][3][4] that there is a lack of knowledge on the correct application of bucking lengths and appropriate C_{my}-factors, when the beam-column formulae is used for buckling checks of cut-out-members. This refers to COM and ECM similarly.

Regarding the cut-out-method (COM), there are currently two main interpretations (called COM/1 and COM/2 in the following). A more conservative approach, COM/1, considers the effect of local imperfections by an additional buckling check based on member length and second-order end-moments. This is in accordance with EN 1993-1-1. The calculation of the C_{my}-factor for COM/1 is commonly based on the first-order shape of the bending moment diagram between the member ends. The second approach, COM/2, considers the effect of local imperfections by an additional buckling check based on the non-sway buckling length and on second-order end-moments. The calculation of the C_{my}-factor is commonly based on the moment diagram between member ends similarly as for COM/1. However, there is no more a physical interrelation between the non-sway buckling length and the shape of the bending moment diagram. But, in subsequent investigations it will be shown that the method COM/2 works well compared to second-order elasto-plastic and second-order yield zone analyses.

Considering the correct application of the C_{my}-factor for the ECM it has been shown by Lechner [5][6] that there are certain relationships between the buckling lengths and the corresponding first-order bending moment diagrams. It has been explained on the basis of numerical studies that the most critical part of the bending moment diagram is the part of the moment diagram between two consecutive inflection points of the lowest buckling mode where the maximum moment is found. Furthermore, it has been shown by interaction diagrams, that for cases where the bending deformation is coincident with the first eigenmode, exact “classical” equivalent members are found. In cases where the inflection points of the bending deformation (points with zero moment) are in good accordance with the inflection points of the first buckling mode good approximation with the ECM can be found. It has been verified, that more complex cases can be solved in a similar way. The following four figures, Fig’s 2a, 2b, 3a, 3b, illustrate the proper application of the ECM to approximate and more complex equivalent members. Although C_{my}-factors depend on slenderness and compression force [2], the application of constant C_{my}-factors can be verified for the investigated cases. Even in frame analysis, good results could be achieved by the ECM.

Fig. 1: Definitions for cut-out-members COM/1 and COM/2.
Figure 2: Application of ECM for approximate (2a) and more complex (2b) beam-columns.

Figure 3: Application of ECM for approximate beam-columns, (3a) and (3b).
An alternative approach of the cut-out-members is also provided in BS 5950-1:2000 by the Amplified Sway Method (BS-ASM). There, the first-order moments are split into two parts. The sway moments are amplified due to the sway-effect and the non-sway moments are reduced by the $C_{my}$-factor for the moment diagram along the column length. The non-sway buckling length should be taken. The application of COM/1/2, ECM and BS-ASM will be illustrated in the parametric study in section 3.

### 3 FRAMES

The following illustrations, Fig’s 4 to 6, are intended to show the behaviour of the different design methods for flexural buckling of steel frames according to EN 1993-1-1. Six specific design methods are investigated by a parametric study. It will be shown to which extent the different characteristics of sway and loading non-sway loading of sway frames will be properly accounted for by the design methods. As mentioned in section 1, two different sway frames with two or three different load-cases are investigated. All frames are built up of RHS 200/100/10 sections steel grade S 235, meaning that beam and column have the same bending stiffness. The overall frame dimensions are set to a ratio that in all critical sections $M_p$ is reached at once due to external loading. The effects of imperfections were not regarded in the choice of the external loading. All numerical simulations are based on elasto-plastic behaviour (Class 2).

**GMNIA:** nonlinear Finite-Element-analyses including first eigenmode-conform imperfections ($L/1000$), residual stress (corner +0.5*$f_y$, faces -0.167*$f_y$) and elasto-plastic stress-strain relationship.

**2O EP-analytical:** elasto-plastic second-order analyses with equivalent global and equivalent local imperfections as specified in EN 1993-1-1, analytical cross-section capacity.

**ECM:** equivalent column method according EN 1993-1-1 with sway-buckling length and first-order moments without any imperfections. The cross-section check is based on the EN 1993-1-1-capacity.

**COM/1:** cut-out-member with second-order end-moments and full member length. Equivalent imperfections are taken as specified in EN 1993-1-1 with initial sway and local imperfections, see Fig. 7. $C_{my}$ is based on member length.

**COM/2:** cut-out-member with second-order end-moments and non-sway buckling length. Equivalent imperfections are taken as specified in EN 1993-1-1 with initial sway and local imperfections, see Fig. 7. $C_{my}$ is based on member length.

**BS-ASM:** Amplified Sway Method acc. BS 5950-1, member-check with EN 1993-1-1, Method 2.

All calculations were performed numerically either by the ABAQUS-software [1] or by Matlab-frame analysis tools. These Matlab-tools enable parametric elasto-plastic second-order plane-frame analyses and subsequent buckling checks according to EN 1993-1-1, Method 2. The cross-section capacity for the methods 2O EP-analytical, COM/1/2 and the Amplified Sway Method is calculated with the analytic formulae. The design checks done by the ECM are based on the section capacity of the EN 1993-1-1 formulae. In all illustrations the buckling checks are limited by the cross-section capacity. Admittedly, in some cases the beam capacity is decisive.
Figure 5: Hinged sway frames with half sway and half non-sway load (5a) and pure non-sway load (5b).

Figure 6: Clamped sway frames with pure sway (6a) and pure non-sway load (6b).
Formula used for cut-out-members designed by COM/1 or COM/2:

$$\frac{N}{k_{y,non-sway}N_{pl}} + k_{y,non-sway} \frac{M_{y}^{II}}{M_{pl,y}} \leq 1$$

$M_{y}^{II}$ is the maximum second-order end moment of the critical column.

Formulae which are in use for the Amplified Sway Method based on BS 5950-1:2000:

$$\frac{N}{k_{y,non-sway}N_{pl}} + k_{y,non-sway}C_{my,member}M_{y,non-sway} \leq 1$$

$$M_{y}^{II} = k_{amp}M_{y,sway} + C_{my,member}M_{y,non-sway}$$

$$k_{amp} = \frac{\Lambda_{cr}}{\Lambda_{cr} - 1}$$

(4)

$C_{my,member}$ is determined from the moment distribution between the member ends. $M_{y,sway}$ considers the effects of initial sway deformation and horizontal forces. $M_{y,non-sway}$ only considers the bending moments due to non-sway loading. All bending moments are first-order internal moments. The second-order amplification factor $k_{amp}$ is herein calculated by the exact values $\Lambda_{cr}$ of the sway system. BS 5950-1 specifies – if $\Lambda_{cr}$ is less than 4.0 – second-order elastic analysis should be used to allow for sway.

4 EFFECT OF LOCAL IMPERFECTIONS

In frame analysis, it is not always obvious which direction of local imperfection is most critical. Therefore, in the herein presented parametric studies two directions of local bow imperfection have been evaluated for the columns. The most critical result was taken for the interaction diagrams shown in section 3 before. In Fig. 7 the investigations which have been performed for the frame study are shown in detail for the case of the clamped sway-frame with pure sway-load. It is shown, that for cases with high axial load the negative local imperfection $e_0$ is decisive. It is illustrated that local bow imperfections have to be included according to EN 1993-1-1 in COM/1/2 when the following requirement is met:

$$\overline{\lambda} > 0.5 \sqrt{\frac{M_{y}}{N}}$$

(5)

$\overline{\lambda}$ is the in-plane slenderness based on member length and $N$ is the compressive force.

![Figure 7](image-url) Study of the effect of local column imperfections at clamped sway frame under sway load.
5 FRAMES – DESIGN BY MODIFIED EQUIVALENT COLUMN METHOD

During the study of sway and non-sway second-order effects in sway frames, the application of an amended equivalent column method was discovered which achieves rather accurate results – even for frames with non-sway loading. As shown in Fig. 8a, a sway frame with sway loading can be well designed with the ECM as described in section 2. To verify this, the equivalent member was also analysed by GMNIA, which is in good accordance with the GMNIA-result of the frame. However, when loading the sway-frame with non-sway load, the underestimation by the ECM lead to inefficient design. This led to a new proposal where the first-order bending moments are separated into their sway, $M_{\text{sway}}^I$, and non-sway, $M_{\text{non-sway}}^I$ parts in the buckling check. Imperfections need not to be accounted for, as in ECM. The ECM-buckling check with the formulae of EN 1993-1-1, Method 2 would be extended to equation (6).

$$\frac{N}{\chi_{,\text{sway}} N_{\text{pl}}^y} + k_{,\text{sway}} \frac{M_{\text{sway}}^I}{M_{\text{pl},y}^I} + k_{,\text{non-sway}} \frac{M_{\text{non-sway}}^I}{M_{\text{pl},y}^I} \leq 1$$  \hspace{1cm} (6)

$$k_{,\text{sway}} = C_{\text{my,sway}} \left(1 + 0.6 \frac{N}{\chi_{,\text{sway}} N_{\text{pl}}^y}\right) \leq C_{\text{my,sway}} \left(1 + 0.6 \frac{N}{\chi_{,\text{sway}} N_{\text{pl}}^y}\right)$$  \hspace{1cm} (7)

$$k_{,\text{non-sway}} = C_{\text{my,non-sway}} \left(1 + 0.6 \frac{N}{\chi_{,\text{non-sway}} N_{\text{pl}}^y}\right) \leq C_{\text{my,non-sway}} \left(1 + 0.6 \frac{N}{\chi_{,\text{non-sway}} N_{\text{pl}}^y}\right)$$  \hspace{1cm} (8)

The moment factors $C_{\text{my,sway}}$ and $C_{\text{my,non-sway}}$ are determined according to the ECM as defined in section 2. In figures 8b and 9 results of a parametric study with above formula (6) are illustrated. It is verified for the different frame systems that this modified ECM leads in general to conservative results. These results are considerably improved in comparison to the ECM for non-sway loading of frames.
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CONCLUSION

To conclude it can be stated, that current EN 1993-1-1 specifications for frame design have been verified. Second-order methods are very accurate. Based on the proper choice of the moment-factor and the buckling length the cut-out-method and the equivalent column method have been checked. It has been found out, that a modified equivalent column method achieves good results for non-sway load of frames.

REFERENCES